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Mark and Lack: On Zero[1]

*[**Editorial note: this manuscript file will be replaced by a definitive pdf in early 2013**]*

[150] Epistemology breaks away from ideological recapture [*reprise*], in which every science comes to mime its own reflection, insofar as it excludes that recapture's institutional operator, the notion of Truth, and proceeds instead according to the concept of a mechanism of production, whose effects, by contrast, one seeks to explain through the theory of its structure.

What of an epistemology of logic?

The representation of this discipline within the network of ideological designators presents it as something foreign to the real, as a discourse that presupposes the positing [*position*] of Truth rather than the construction of an object. This is what Frege abruptly declares when he likens a proposition to a proper name whose reference, or denotation, is the True, or the False. It follows from this that logic incessantly coordinates as many linked inscriptions as necessary in order for it to pass from one invariable name-of-the-True to another: thus logic here becomes *the scriptural indefiniteness of truth's civil status* [*l'indéfini scriptural d'un état civil de la vérité*].[2]

On the basis of all this one can in fact demonstrate – as Jacques Lacan and Jacques-Alain Miller undertake to do – that, as something which can be known under several names, the True falls beneath its names, while nonetheless preserving its civil status through the iteration that, at its perpetual birth, has us ceaselessly registering its new anonymous names. The nominal movement, the repetitive compulsion that, in the chain of propositions, unravels our disbelief in the True's common patronym, marks nothing but the lack over which this movement glides without resistance or success.[3]

To this twofold process (preservation of the True; convocation and marking of lack), we will oppose the stratification of the scientific signifier.

In our view, *both* Frege's ideological representation of his own enterprise *and* the recapture of this representation in the lexicon of Signifier, lack and the place-of-lack, mask the pure productive essence, the [151] positional process through which logic, as machine, lacks nothing it does not produce elsewhere.

The logic of the Signifier[4] is a metaphysics: a representation of representation, an intra-ideological process and progression.

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[1] TN: First published as Alain Badiou, 'Marque et manque: à propos de zéro', CpA 10.8 (winter 1969), 150-173. Translated by Zachary Luke Fraser with Ray Brassier.

[2] Cf. Gottlob Frege, 'On Sense and Nominatum', in *Readings in Philosophical Analysis*, ed. Herbert Feigl and Wilfrid Sellars (New York: Appleton-Century-Crofts, 1949), 85-102. 'Every declarative sentence, in which what matters are the nominata of the words, is therefore to be considered as a proper name; and its nominatum, if there is any, is either the True or the False' (91). '[A]ll true sentences have the same nominatum, and likewise all false ones' (92).

[3] TN: The French reads: '*Le mouvement nominal, la compulsion répétitive où se déploie l'impuissance à croire tenir jamais le patronyme usuel du Vrai, c'est la marque même, dans la séquence liée des propositions, de ce qui n'est qu'un manque sur quoi elle glisse sans résistance ni succès.*'

[4] By 'logic of the Signifier', we mean here the system of concepts through which the articulation of the subject is conceived: Lack, Place, Placeholder, Suture, Foreclosure, Splitting. These concepts have been produced by Jacques Lacan and we acknowledge a definitive debt to him even as we engage in the process that circumscribes their use: this is the critical procedure.

The thesis we are defending here aims only at delineating the impossibility of a logic of the Signifier that would envelop the scientific order and in which the erasure of the epistemological break would be articulated.

I Triple Articulation of the Logical Process

The theory of logic pertains to the modes of production of a division in linear writing or inscription: the dichotomy of a structured set of statements which have been 'introduced' into the final mechanism as an (already processed) raw material.

The immediate consequence of this is that the sole requirement that the functioning of the mechanism must satisfy is that ultimately something must be *cut* in an effective fashion: the inscriptions [*écritures*] must be mechanically separated into two disjoint [*disjointes*] classes, which, by allusion to the mechanism most frequently employed, are called the class of derivable statements and the class of non-derivable statements respectively.

The classical definition of the *absolute consistency* of a system, according to which at least one well-formed expression not be derivable within it, designates precisely this minimal requirement. Its infraction would be tantamount to considering a mechanism that produces nothing at all – production in this instance being just the effective division of those materials on which one is operating.

On closer inspection, it becomes clear that this final division implies the successive operation of three intricated mechanisms: before they can be allocated [to the class of derivable or non-derivable statements], the syntagms must be *formed,* then *sorted*, since no derivational system is capable of submitting all of them to its principle of division. (This just means that every specialized machine has an input [*entrée*] into which only specific and previously processed materials can be introduced).

We must therefore distinguish the mechanisms of *concatenation*, *formation*, and *derivation*.

Any occlusion of the autonomy of the second mechanism – relative to the third – entails losing the very essence, i.e. the productive function, of the logical process.[5] And nothing is more important than to traverse the machineries of logic in their proper order.

(a) Concatenation: The absolutely primary raw material of the logical process [152] is supplied by a particular sphere of technical production: writing. This consists of a stock of graphic marks, separable and indecomposable, forming a finite (or at most, denumerable) set, a set we will call the 'alphabet'.

The first mechanism 'receives' these marks, from which it composes *finite sequences* (linear juxtapositions which may include repetitions). It is set up to produce all the finite sequences of this sort, and so it is these that we find in the output of this mechanism. Let S be this product.

(b) Formation: The second mechanism operates on S, so as to effect, step by step, a *perfect dichotomy*, one that separates without remainder those sequences the machine 'accepts' from those that it rejects. We will call those sequences accepted by the machine 'well-formed expressions', and the others 'ill-formed'.[6]

The operators (the 'components') of this mechanism are *the rules of formation*, which prescribe certain configurations as acceptable concatenations: thus, for example, the machine known as 'the predicate calculus with equality' will accept the sequences *I(x, x)* and *not-I(x, x)*, but will reject the sequence *x(I, x)*.

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[5] The privileged operator of this occlusion is the concept of meaning or sense [*sens*], to which both the origination of the True (derivability) and the rejection of non-sense (syntax-formation) refer back.

[6] That the division be without remainder means that given any inscription whatsoever (i.e. a finite sequence of signs of the alphabet), there exists an actual procedure that permits one to determine *unambiguously* whether the expression does or does not conform to the rules of the syntax.

For classical logics, this syntactic property can be made the object of a recursive demonstration over the number of parentheses in the expression. Cf. Stephen Cole Kleene, *Introduction to Metamathematics* (Amsterdam: North Holland Publishing Co., 1964), 72 and *passim*.

Through a dangerous semantic laxity, the rejected statements are often called 'non-sense'.

The set of rules of formation constitutes the syntax.

Let us note straightaway that if, as Gödel's celebrated theorem indicates, the final dichotomy (that of the third mechanism) cannot, for a 'strong' machine, be effectuated without remainder[7] – since there are always *undecidable* statements – the very possibility of this result *presupposes the existence of a dichotomic mechanism that leaves no remainder*: the one which supplies the demonstrative mechanism with its raw material, the well-formed expressions. Only on the condition of a perfect syntax can we summon derivation's aporias.

The split signifying order, marked by what it lacks, can be exhibited [153] only in its difference from an autonomous order that is indeed closed, which is to say, entirely decidable (the order of the formation of syntagms). In this sense, we cannot maintain that scission or compulsive iteration is the inevitable price of closure. What must be said, instead, is that the existence of an infallible closed mechanism conditions the existence of a mechanism that can be said to be unclosable, and therefore internally limited.

The exhibition of a suture presupposes the existence of a foreclosure.

Now, theoretical anticipations aide, what needs to be remembered at this point is that what we find in the output of the syntactic mechanism is the set of well formed expressions, which we will call E.

(c) Derivation: The third mechanism operates on E and, in general, is set up to produce:

1: A perfect dichotomy between Theses (or derivable statements) and non-Theses (non-derivable statements).
2: A certain type of functional relation between these two divided halves.

This second condition is crucial. If dichotomy was the only requirement then the classical logical mechanisms (the formalization of arithmetic, for example) would be flawless, since all of these mechanisms do indeed separate well-formed expressions without remainder into the derivable and the non-derivable, i.e. into theses (T) and non-theses (NT).[8]

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[7] A strong machine is one capable of partitioning the inscriptions of recursive arithmetic.

Note that there exists a weak but *perfect* logical mechanism: the Propositional Calculus. This system is indeed:

– Consistent in every sense of the term;
– Decidable (for every well-formed expression, it can be mechanically known whether or not it is derivable);
– Complete (every well-formed expression is either derivable or such that to add it to the axioms would render the calculus inconsistent);
– Categorical (all of its models are isomorphic).

The mere existence of this Calculus presents several problems for the Logic of the Signifier, since it contains nothing – not even an empty place – that would attest to a lack. In all rigour, this system lacks nothing; nor does it mark the nothing of which it is already too much to say that it is even lacking.

One could say that the perfection of the Propositional Calculus provides the intra-logical, differential referent for the relative 'imperfection' of other systems.

[8] It is an entirely *different* question to determine whether or not there exists for every well-formed expression a mechanical (effective) procedure that would allow us to know 'in advance' (i.e. without having to carry out the derivation) whether or not it is derivable.

The existence of such a procedure defines the *decidability* of a system. We know (Church, Kleene) that sufficiently strong logical mechanisms are generally undecidable.

We should not confuse *the decidability of a system* with the existence or non-existence of a statement such that neither it nor its negation are derivable. The problem of the existence of an undecidable statement is not a problem of decidability, but a problem of *completeness*.

An undecidable statement, such as the one constructed by Gödel, is obviously not a statement that would be neither neither provable [*démontrable*] nor unprovable (which would be meaningless). On the contrary, the heart of Gödel's proof consists in the demonstration that such a statement *is not* provable. It is therefore clearly assigned to one of the two halves.

An undecidable statement is not the remainder of a cut, but a statement which is such that neither it nor its negation is derivable. Such a statement is certainly *irrefutable* (refutation = proof of the negation). But it is explicitly unprovable. There is indeed a division without remainder between derivable and non-derivable statements – but both Gödel's statement and its negation end up in *the same division*.

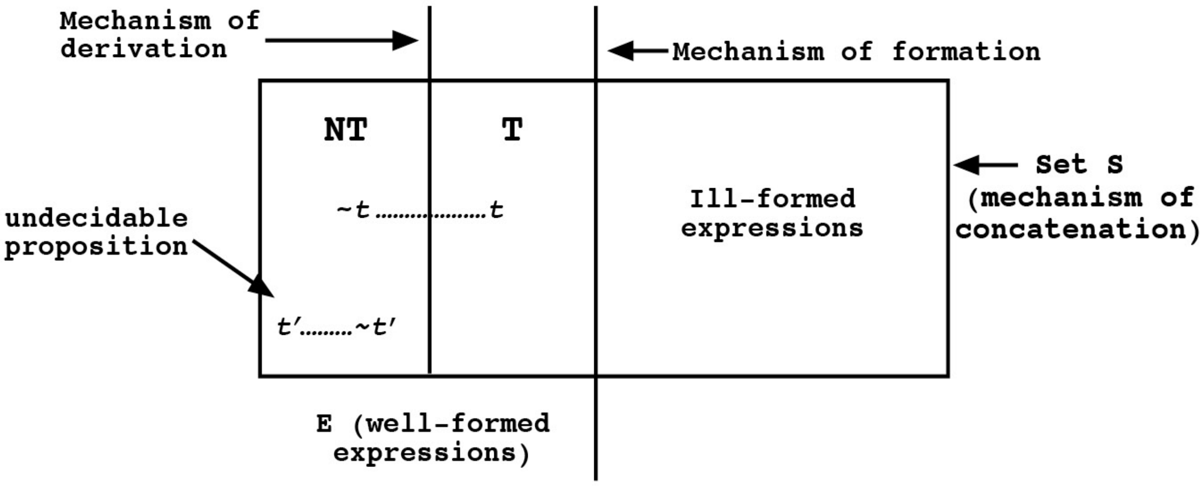
Everything depends here upon a special syntactic operator and the structure which it governs: the operator of negation. [154]

We cannot therefore take Gödel's theorem to mean that every dichotomy leaves a remainder, or that every duality implies a disjoint third term, one that would be de-centred relative to the rule that internally orders each term of the pair. This (common) reading of the theorem is a metaphysical import . In reality, the problem pertains to the particular structural conditions imposed on the third logical mechanism, over and above its separative function – this is summed up above in our condition 2.

What is required is as follows: in the alphabet, there needs to be an operator (it can be negation [symbolised as ~] or any other: the intuitive meaning of negation is an obstacle here) such that if a statement belongs to one division ($t \in$ T or $t \in$ NT), then the statement obtained by applying to it the operator (i.e. ~)[9] will be in the other division ($\sim t \in$ NT or $\sim t \in$ T).

What is originally at issue here is not the cut as such, but a function relating the separated halves. The Gödelian limit does not bear on the dichotomy as such. Rather, it concerns the unity-of-correspondence of the disjoint parts.

Gödel's statement signifies: let there be a functional relation that sends each statement to its negation (t ..... $\sim t$). There is no effective dichotomy that cuts through *all* of those relations.



A system may be decidable and yet incomplete: there then exist in it (undecidable) statements concerning which it is possible to 'decide' in advance, through an actual procedure, that they are neither derivable nor refutable. The converse, however, is not true: an important meta-mathematical theorem ties the undecidability results (Church) to the incompleteness results (Gödel). If a (sufficiently strong) formal system is undecidable, then it is either inconsistent or incomplete.

[9] In conformity with common usage, we will use the symbol ~ to denote the function of negation throughout the remainder of this exposition.

One might hope to expel *from T* (the set of derivable statements) all the relations *t* ..... ~t; otherwise, the system would be inconsistent. But one can then go on to show that some of these relations will always remain in NT: precisely those which concern undecidable statements.

What we are faced with here is a tearing of structure rather than a dichotomy. The key to the limitation [*limitation*] follows paradoxically from the fact that the separating mechanism is forced *not to be* perfect, and thus forced to preserve the concept of a reversible *relation* between the two halves. As a result, this limitation, far from attesting that the space produced by the division bears the trace of the tear that caused it, shows rather that one cannot indefinitely produce the sign of the latter within it; that in certain places the trace is effaced; that a strong mechanism [155] necessitates a complete division in rejection it effects, in each of its parts, of certain marks of the old Whole [*Tout*].

The undecidable is not the suturation of lack but *the foreclosure of what is lacking* through the failure to produce, within what is derivable, the whole of the non-derivable as negated.

The limitation means: that there exists at some point, between the parts T and NT, a *distance without concept*: one that delineates, in the space of non-theses, a statement whose negation cannot be inscribed within the space of theses, and which is therefore un-related to this space. Gödel's theorem is not the site of separation's failure, but of its greatest efficacy.[10]

If, therefore, the theorems of 'limitation' result from the conditions of imperfection assigned to the dichotomic mechanism, we must reconfigure [*remanier*] the concept of the latter so as to incoporate those conditions. We will then say:

Logic is a triply articulated system (concatenation, syntax, derivation) that produces a terminal division in linear inscription such that, given a suitable syntagm, we should be able:

i) To allocate it to one of the two halves (T or NT);
ii) To construct a syntagm obtained mechanically from the first by the addition of a functor (generally called negation), such that if the first is in one division, the second will be in the other.

Condition (i) is ideally[11] satisfied by classical mechanisms (set theory or the formalization of arithmetic). The second is satisfied only by weak mechanisms: a strong mechanism cuts *all too well*.

II Nullity of the Thing – Identity of Marks

This description of the logical mechanism allows us to question the construction of the concept of suture in this domain, and allows us to precisely determine the meta-theoretic function of the zero.

Let us declare our theses at the outset:

1) The concept of identity holds only for marks. Logic never has recourse to any

[10] We will abstain from any attempt to decipher the status of the hiatus between intuitionism and formalism in Gödel's theorem. On this point, see our appendix on Smullyan's demonstration, and our critique of the concept of limitation.

[11] 'Ideally' because although it is true that every well-formed expression is either in T or NT, the existence of an 'effective' (recursive, algorithmic) procedure that would allow us to determine into which of these two classes it falls is shown to be impossible in many instances. This is the problem of the decidability *of the system* (cf. note 8).

self-identical *thing*, even when 'thing' is understood in the sense of the object of scientific discourse.

2) The concept of truth is an ideological designator, both recapitulating and concealing the scientific concepts of selection and division. It designates globally what is, itself, a differentiated mechanism.
3) The zero is not *the mark of lack* in a system, but the sign [156] that abbreviates the *lack of a mark*. Or rather: it is the indication, within a signifying order, that there is an inscription present in the rejected division of another order.
4) The logico-mathematical signifier is sutured only to itself. It is indefinitely *stratified*.
5) In logic, a lack that *is* not a signifier *has* no signifier: it is foreclosed.
6) The signifier in general is not articulated to lack through the concept of suture, whose purchase demands that the signifier satisfy a certain condition. And the construction of that condition is not the task of psychoanalysis but of historical materialism: only the *ideological* signifier is sutured.

Like Lacan's accounts of Gödel's theorem and the semantics of implication, Jacques-Alain Miller's discussions of Frege[12] and Boole are ambiguous in that they combine, simultaneously and indistinctly, what pertains to the effective construction of a logical mechanism with what pertains to the (ideological) discourse through which logicians represent their constructions to themselves.

Consequently, we should be wary of comprehending *within* the logical process itself any translation of the signs' connective agency back into the lexicon of subsumption. This notion, enclosed in the (specular) referential relation, like the related notion of denotation, masks the strictly functional essence of the mappings [*renvois*] at work inside logical mechanism.

Nothing here warrants the title of '*object*'. Here the thing is null: no inscription can objectify it.

Within this mechanical space, one finds nothing but reversible *functions* from system to system, from mark to mark – nothing but the mechanical dependencies of mechanisms. Semantics itself enters into logic only insofar as it operates between two logico-mathematical signifying orders, and on condition that the functions of correspondence between these two orders are themselves logico-mathematical.[13]

Neither thing nor object have the slightest chance here of acceding to any existence beyond their exclusion without trace.

It follows that the Leibnizian requirement of self-identity, which is necessary in order to preserve truth, is intra-logical (theoretical) only insofar as it pertains to the identity of marks. It postulates, on the basis of an inaugural confidence in the permanence of graphemes [*graphies*], that there exists an 'identical' application of the signifying order to itself, one that preserves its structure. [157]

Moreover, it is science as a whole that takes self-identity to be a predicate of marks rather than of the object. This rule certainly holds for those *facts of writing* proper

[12] Cf. Jacques-Alain Miller, 'La Suture', CpA 1.3.

[13] We believe Alonzo Church is right to identify Semantics with Syntax in the last instance (cf. his *Introduction to Mathematical Logic* (Princeton: Princeton University Press, 1956), 65: 'These assignments of denotations and values to the well-formed formulas may be made as abstract correspondences, so that their treatment belongs to theoretical syntax').

Semantics becomes *logical* (scientific) only when it is *the syntax of the difference between syntaxes.*

TN: For a more extended treatment of the relation between syntax and semantics, see Alain Badiou, *The Concept of Model* [1969], ed. Zachary Luke Fraser and Tzuchien Tho, trans. Zachary Luke Fraser (Melbourne: re.press, 2007).

to Mathematics. But it also holds for those *inscriptions of energy* proper to Physics. As Bachelard has admirably demonstrated, the only properly physical rule of substitution concerns artificial operators: 'The principle of the identity of instruments is the true principle of identity in every experimental science.'[14] It is the technical invariance of traces and instruments that subtracts all ambiguity from the substitution of terms.

Thus determined, the rule of self-identity allows of no exceptions and tolerates no evocation of what evades it, not even in the form of rejection. What is not substitutable-for-itself is something radically unthought, of which the logical mechanism *bears no trace*. It is impossible to turn it into an evanescence, a shimmering oscillation, as Frege does when he phantasmatically (ideologically) convokes then revokes the thing that is not self-identical in order to summon the zero. What is not substitutable-for-itself is foreclosed without appeal or mark.

Yet a homonymous predicate can indeed be constructed within logical systems: there exist 'calculi of identity' in which non-identity is marked.

In order to avoid the slippages of language, let us agree to give the name 'equality' to such a homonymous predicate, which we will denote by $I(x,y)$ (ordinarily, this would be read as: x is identical to y).

We are going to show that the customary homonymy dissimulates a relation of presupposition that exhibits, again, the priority of the foreclosed [*du forclos*].

Consider for example a first-order calculus (one in which it is impossible to quantify over predicates): the predicative *constant* of equality I is implicitly defined by way of two axioms:[15]

- $I(x,x)$ (Total reflexivity)
- $I(x,y) \supset [A(x) \supset A(y)]$

It might be thought that the axiom of reflexivity formulates within the inscriptions of the calculus (in the output of the syntactic mechanism) the fundamental self-identity of any letter at all. But this is not the case: what we have agreed to call the self-equality of a variable *is not* the self-identity of every mark. The best proof of this is that this equality allows for the construction of its negation: $\sim I(x,x)$ is a well-formed expression of the system, a legible expression.

Yet it would be wrong to imagine that $\sim I(x,x)$ (which should be read as: x is not equal – or identical – to itself) *marks* within the system, or positions within the mechanism, the unthinkable non-self-identity of the sign, and that such a (well-formed) expression organizes the suturing of the unthinkable to the calculus. On the contrary, far from marking the unthought, the [158] signifying existence of $\sim I(x,x)$ presupposes its functioning *without a mark*: it is necessary that one be *unable* to conceive that x, qua mark, is 'other' than x – the same mark placed elsewhere – in order for this statement to be logically produced. The mere convocation-revocation of x's non-self-identity, the shimmering of its self-differing, would suffice to annihilate the scriptural existence of the entire calculus, and particularly of expressions such as $\sim I(x,x)$, in which x occurs twice.

The production of the logical concepts of equality and self-inequality presupposes the foreclosure of what is scripturally non-self-identical. The lack of the equal is built upon the *absolute* absence of the non-identical.

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[14] Gaston Bachelard, *L'Activité rationaliste de la physique contemporaine* (Paris: PUF, 1951), 5.

[15] In a second order calculus, in which one can quantify over predicates, equality would be defined explicitly, in conformity with the Leibnizian doctrine of indiscernibles, which is restricted here to the order of signs: two individual variables falling without exception under all the same predicates can be substituted everywhere, since there is nothing to mark their difference. In classical notation:

$$I(x,y) =_{df} (\forall a)[a(x) \supset a(y)].$$

No doubt the structure of a calculus of identity generally implies the derivation of the thesis ~~I(*x,x*):[16] it is false that *x* should not be equal to *x*. But as far as lack is concerned, this 'negation' marks nothing but the rejection of (or presence in) the other division (that of non-theses) of the statement ~I(*x,x*), which has been produced identically by the syntactic mechanism. No absence is convoked here that would be anything but the allocation to one class rather than to its complement – according to the positive rules of a mechanism – of what this mechanism receives from the productions of another.

This allows us to relate, without ideological infiltration, the concept of identity to the concept of truth.

Nothing transpires here of the thing or its concept.

But the statement 'truth is',[17] a purely expedient designation of an operational complex, signifies, so far as identity and equality are concerned:[18]

Identity: The relation that logic bears towards writing is such that it receives from the latter only those marks that have been certified by the signifying chain as capable of being substituted for themselves everywhere. In truth, this means any mark whatsoever, whose invariable recognition is rooted in the (external) technique of graphemes.

Equality: There exists a signifying order (a mechanism of derivation), whose selective constraints are such that the statements I(*x,x*) and ~I(*x,x*) are sent to two different divisions.

If, from a perspective that is more strictly that of mathematical logic, one wishes to consider the product of mechanism-3 as the set of derivable theses, one can say: the mechanism is set up in such a way that it produces I(*x,x*) and rejects ~I(*x,x*).

But these two inscriptions have already been produced in *the same division* (that of well-formed expressions) by a mechanism-2 (a syntax). Only on this basis is it possible to give any sense to the rejection of one of the two by the mechanism of derivation.

What is not equal-to-itself is only excluded here on condition of having to be placed within an autonomous signifying order, sedimentarily organized 'beneath' the one which no longer has a place for it.

To maintain at all costs, in this point, the correlation between the equal-to-itself and the true would be to say: truth is the system of constraints [159] which differentiate the mechanism-3, producing the single statement I(*x,x*) from mechanism-2, which produces I(*x,x*) and ~I(*x,x*) simultaneously.

The equal-to-itself as salvation of truth comes down to no more than a difference, thanks to a withdrawn effect [*par effet retiré*], between syntax and derivation, between raw material and product. More precisely: a difference between two selection mechanisms, where the second is finer than the first.

III Mark of Lack or Lacking Mark?

We can now hazard the Zero.

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[16] TN: The *Cahiers* text has '~I(*x,x*)' rather than '~~I(*x,x*)', but it is clear from the context that this is a misprint. As Badiou remarks, it is *false* that *x* should be unequal to *x*. The negation in the formula should therefore be double.

[17] TN: This is a reference to the article 'Suture', where Miller writes, after Leibniz: 'Truth is. Each thing is identical to itself' (CpA 1.3:43).

[18] TN: The French reads '*Mais "la vérité est", pure désignation commode d'un complexe opératoire, signifie, s'il faut y pointer l'identité et l'égalité:*'.

To introduce it by way of a definition, the symbol zero is an abbreviator, standing for an inscription produced by a mechanism-2.[19] It is an *abstraction* (the construction of a one-place predicate) over a relation.

Let us provisionally adopt Frege's 'set-theoretical' language.

Given any relation between individual variables, say R(x,y), it is possible to construct the class of all x satisfying R(x,x), and to consider membership in this class as a property or predicate: the predicate 'to be linked to itself by the relation R'. One has thereby carried out *the abstraction of reflexivity over the relation R*.

Let us denote this new predicate A*r*•R. A*r*•R(x) 'signifies': x has the property of being linked to itself by the relation R.

These considerations, which rest on an 'intuitive' concept of class, must now be abandoned, since they are foreign to the logical mechanism: they pertain only to the ideological pedagogy of the system.

In truth, all we have is a syntactic rule immanent to M_2,, which allows us to:

(a) Construct, on the basis of a *two*-place predicate (R), the accepted inscription A*r*•R.

(b) To treat this inscription exactly like any other *one*-place predicate (which, for example, allows us to write A*r*•R(x), etc.).

Abstraction here is therefore a rule that allows the mechanical formation of a one-place predicate from a two-place predicate.

Naturally, this abstraction can be carried out on the relation I(x,y), which we have called the relation of identity. Since I(x,x) is precisely one of the axioms of the calculus of identity, the M_3 of this calculus will trivially derive the statement ($\forall x$)(A*r*•I(x)), i.e.: every x is linked to itself by the relation I.

But the abstraction of reflexivity can just as well be carried out over the relation of inequality, ~I(x,y), since this inscription is produced by M_2.

We thereby obtain *one* of the possible definitions of th*e zero predicate*:

$$0 = \mathrm{A}r\bullet\sim\mathrm{I}$$

[160] 0(x) could then be read as: x is a zero, it has the property of not being equal to itself.

Satisfying 0(x) – being a zero – will in no way prevent the sign x, or the sign 0, from being everywhere substitutable for themselves: they remain identical, even where they support or name non-equality (or non-identity) to self.[20]

To say that the zero, so defined, 'aims at' ['*vise*'] a non-self identical object, or that it is the predicate of the void, convokes a metaphysical reading of Being and its Plenitude precisely at that point where only substitutions of inscriptions obtain.

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[19] We will henceforth designate as M_1, M_2 and M_3, the mechanisms of concatenation, of syntax (of the predicate calculus), and of derivation (idem) respectively.

[20] Some might find it surprising that we have here constructed the zero not as a term but as a *predicate*.

But it is Jacques-Alain Miller whom we must question about his reiteration of Frege's failure to distinguish between individual and predicative variables. For Frege, certainly, a predicate is a term. But this position is untenable, for it gives rise to Russell's paradox, which would eventually ruin Frege's formal arithmetic.

Miller's text, however, does not integrate the *theoretical inconsistency* of Frege's construction of number into its own *metatheoretical employment* of the latter. There results an epistemological equivocation, dissipated only if one distinguishes the level of functioning proper to each *mention* of Frege's (confused) text. Namely:

(a) A theoretical attempt to construct the finite cardinals.

(b) The theoretical errors in this attempt (the non-stratification of variables).

(c) The ideological re-presentation of the theoretical [*du théorique*] (denotation, concept, number of concept, etc.).

(d) The ideological re-presentation of these theoretical errors (theory of the zero).

For the inscription ~I(x,x) does not occupy the place of anything else; nor does it mark the place of a nothing.

As for the zero, it occurs at every place occupied by that to which scriptural convention has declared it equivalent: Ar•~I. It is positively constructed by M_2.

We will call mechanism-4 the logical system that adds to M_3 the predicative *constant* (the proper name) 0, as it has been defined above. Of what lack could this addition be the mark, in the signifying order thus designated?

M_3, as we have seen, *rejects* the inscription ~I(x,x), and *derives* the inscription I(x,x). Must we not consider that the predicate zero *marks* within the non-rejected division of M_4 what has been rejected in M_3? Is it not the predicate satisfied by 'no' term?

In truth, such descriptions are foreign to logical theory. The zero is simply an inscription accepted by M_2 and introduced, along with certain directions for use, in M_4.

If one nevertheless wants to think the zero's link to the non-figuration of ~I(x,x) in the derivation of M_3, a somewhat allegorical use of concepts is necessary. But it is acceptable to say: The zero marks in M_4 (in predicative form) not the *lack of a term* satisfying a relation but rather a *relation lacking* in M_3, the relation ~I(x,x). We must nevertheless add: if the relation can be lacking in M_3, it is *only insofar as it figures in M_2*.

Play of appearances and disappearances between successive signifying orders; never exposed to the convocation of a lack, whether in the object or the thing.

System of differences between systems, ruled by substitutions, [161] equivalences, and withdrawals: *lacking mark, never mark of lack*.[21]

It is not a blank space whose place the zero names, but *the erasure of a trace*: it leaves visible beneath its mark (Ar.~I) *the other* mark (~I(x,x)), as rejected by derivation.

The zero is the mark (in M_4) of a mark (in M_2) that is lacking (in M_3).

On this side of the signifying chain, if the latter is scientific, there are nothing but other chains. If the signifier is sutured, it is only to itself. It is only itself that it lacks at each of its levels: it regulates its lacks without taking leave of itself. The scientific signifier is neither sutured nor split, but stratified.[22] And stratification repeals the axiom by which Miller, in another text,[23] characterized foreclosure: the lack of a lack is also a lack. No, not if that which comes to be lacking was always already marked: then the productive difference of strata suffices to name the interstice. The *halting points* are always prescribed.

IV The Torment of Philosophy

Must we therefore renounce [*annuler*] the concept of suture? It is, on the contrary, a matter of prescribing its function by assigning to it its proper domain.

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[21] TN: '*marque manquante, jamais marque du manque*'.

[22] Ramified calculi (the various instances of the theory of types) attempt to reduce stratification to a single stratum, via the construction of a *logic of stratification* that would 'express' the *stratification of logic*.

The inevitable axiom of *reducibility* indicates a certain failure of this endeavour (see for example, Wilfred Quine, 'On the Axiom of Reducibility', *Mind* 45 [1936], 498-500).

Hao Wang's 'expansive' system, $\sum$, is rather a *constructive traversal* of stratification. It is no less exposed to considerable difficulties concerning the construction of the ordinals. Cf. for example Wang, *A Survey of Mathematical Logic* (Peking: Science Press, 1964), 559ff., especially 643).

For our part, we are convinced that the stratified multiplicity of the scientific signifier, which is inherent to the process of scientific production, is irreducible to any of its orders. The space of marks does not allow itself to be projected onto a plane. And this is a resistance (or limitation) only from the viewpoint of a *metaphysical* want [*vouloir*]. Science wants the transformation-traversal of a stratified space, not its reduction [*rabattement*].

[23] Miller, 'L'Action de la structure', CpA 9:6.

From the fact that there exists a signifying order, namely science (stratified in such a way that no lack is marked in it that does not refer to another mark in a subjacent order differentiated from the first), an exception results. Science does not fall under the concept of the logic of the signifier. In truth, it is the fact that it does not fall under this logic that constitutes it: the epistemological break must be thought under the un-representable auspices of de-suturation.

Accordingly, *there is no subject of science*. Infinitely stratified, regulating its passages, science is pure space [*l'espace pure*], without inverse or mark or place of that which it excludes.

Foreclosure, but of nothing, science may be called the psychosis of no subject, and hence of all: universal by right, shared delirium, one has only to maintain oneself within it in order to [162] be no-one, anonymously dispersed in the hierarchy of orders.

Science is the Outside without a blind-spot.[24]

Conversely, the signifying structure defined by suturation can be designated in its particularity (as that which places lack), primarily as non-science. Thus the concept of suture is not a concept of the signifier in general, but rather the characteristic property of the signifying order wherein the subject comes to be barred – namely, *ideology*.

There is always a subject of ideology, for this is the very mark by which we recognize the latter. Place of lack, splitting of the closed: these are the concepts on whose basis we can elaborate the law governing the functioning of ideological discourse.

We should take the measure of what is at stake here, the possibility of articulating Historical Materialism and Psychoanalysis: the former producing the Schema [*Topique*] of particular signifying orders (ideologies), the latter producing the structures of their efficacy, the laws of entry [*entrée*] and connection through which the places allocated by ideology are ultimately occupied.

When Historical Materialism claims to be able to elucidate subjective enslavement to ideologies on its own, or when psychoanalysis effaces the specificity of the place where it must uncover the mark of lack in the generality of a logic of the signifier, then these disciplines are collapsed and reduced to one another. They become un-stratified: un-scientific.

We need to insist, then, that psychoanalysis has *nothing to say* about science, even if it can teach us a great deal about the scientists who serve it. Through this silence, psychoanalysis negatively determines the signifier of which it speaks, and in which it articulates Desire. Historical materialism provides a positive redoubling of this determination by producing the structural configuration in which ideological agency takes place.

Accordingly, to claim that the science/ideology difference could be effaced through a logic of oscillating iteration, and to nominate [*nommer*] a subject of science, is to preclude the possibility of conjoining, though their very disjunction, Marx and Freud.

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[24] If one wants to exhibit writing as such, and to excise its author; if one wants to follow Mallarmé in enjoining the written work to occur with neither subject nor Subject, there is a way of doing this that is radical, secular, and exclusive of every other: by entering into the writings of science, whose law consists precisely in this.

But when literary writing, delectable no doubt but obviously freighted with the marks of everything it denies, presents itself to us as something standing on its own in the scriptural Outside, we *know* in advance (this is a decidable problem...) that it merely sports the *ideology* of difference, rather than exhibiting its real process.

Those writers who balk at the prospect of taking up mathematics should limit their agendas to the honourable principle of their own productions: to be *ideology exposed*, and thereby irreducibly sutured, even if autonomous.

To deploy the concept of suture in the very place where it is inadequate (mathematics), and to conclude that this concept enjoys a universal legitimacy over discourses by exploiting scientists' conflation of their own activity (science) with its (ideological) re-presentation, is to reflect science in ideology: it is to de-stratify it so as to prescribe to it its lack. [163]

We will call 'philosophy' the ideological region specializing in science, the one charged with *effacing* the break by *displaying* the scientific signifier as a regional paradigm of the signifier-in-itself: this is Plato's relation to Eudoxus, Leibniz's relation to Leibniz, Kant's relation to Newton, Husserl's relation to Bolzano and Frege, and perhaps Lacan's relation to Mathematical Logic.

Science, as we have shown, is that which relates only to itself, the multiple outside. No signifying order can envelop the strata of its discourse.

Whence the recurrent impossibility of philosophy, whose polymorphous historicity attests to the fact that the law of ideology is well and truly operative in it: philosophy transmits and insists the mark of its lack.[25]

And what does it lack? The effacement of the break presupposes the intra-philosophical construction of a concept of science. Philosophy is compelled to mark, within its own order, the scientific signifier as a *total* space. But science, indefinitely stratified, multiple foreclosure, difference of differences, cannot receive this mark. The multiplicity of its orders is irreducible:[26] that which, in philosophy, declares itself science, is invariably *the lack* of science. That which philosophy lacks, and that to which it is sutured, is its very object (science), which is nevertheless marked within the former by the place it will never come to occupy.

We can claim in all rigour that *science is the Subject of philosophy*, and this precisely because there is no Subject of science.

Taking up our invocation of Leibniz once more, this means that in order for ideology to be saved (i.e. the dominant class), the unclosable opening which science tears within it must be *placed* in it. Philosophy consummates itself in this placement.

This is why science and the practice of science will always torment philosophy. Summoning the multiple to its self-sufficiency, the game of science delights us with the lesson of its non-presence (unless it be as a symptom of its own lack) in philosophical discourse. Through science we learn that *there is* something un-sutured; something foreclosed, in which even lack is not lacking, and that by trying to show us the contrary, in the figure of Being gnawing at itself, haunted by the mark of non-being, philosophy exhausts itself trying to keep alive its supreme and specific product: God or Man, depending on the case.

Spinoza said so categorically.[27] Lautréamont too, [164] eulogizing mathematics with a sort of sacred delight:

> O austere mathematics, I have not forgotten you, since your wise lessons, sweeter than honey, filtered into my heart like a refreshing wave (*Maldoror*, Book II).

And Lautréamont, divulging the key to his enthusiasm, adds splendidly: 'Without you in my struggle against man, I might have been defeated.'

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[25] TN: '*la philosophie véhicule et insiste la marque de son manque*.'

[26] This is obviously not to say that regional 'syntheses', transferences [*transferts*], or intrications, are impossible. The history of the sciences thinks the *local connectivity* of strata, and the stratification of this connectivity.

Auguste Comte's greatness resides in his having seen that the multiplicity and hierarchy in the signifying order, whatever displacements and intersections might be engendered in it, were properties inherent to the concept of scientificity.

[27] In a famous text: Spinoza, *Ethics*, Book I, appendix. Man would never have ventured beyond illusion had it not been for this surprising *fact*: mathematics.

In mathematics, there is indeed nothing lacking that is not already signifying [*signifiant*]: marks indefinitely substituted for one another in the complication of their entangled errancy.

Science is the veritable archi-theatre of writing: traces, erased traces, traces of traces; the movement where we never risk encountering this detestable figure of Man: the sign of nothing.

January, 1967

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APPENDIX

Gödel's Theorem and the Alternating Chain of Science-Ideology

With regard to which undertakings can Gödel's theorem be described as a *limitation*? There are, essentially, two:

(I) The (metaphysical) project which, following Hilbert, enjoins every formal system to seal itself around the internal statement of its own consistency.

Once subjected to this injunction, mathematics would no longer expose knowledge to the indefinite abyss in which it stacks its signifying orders: under the aegis of what Husserl called a 'nomological system', it would provide the constitutive charms [*maléfices constituants*] of philosophy with a language that is closed, unique, and self-norming; one which, itself immobile, would efface the wound which was historically opened within the weave of ideology by the *fact* of science.

To force the scientific signifier willingly to occupy the place where it is occluded – this is a nice trick, but one with which this signifier refuses to comply. We will see why.

(II) The project which, by means of the completely controlled reconstruction of a logistical system, claims to exhaust what otherwise presents itself according to the opacity that results from a history: let us call this 'intuitive' arithmetic.

[LINE BREAK]

We have already indicated how the first requirement should be regarded. It admirably illustrates *philosophy's* failure to prescribe to mathematical inscriptions even the unity of a space of existence. It experiences stratification's resistance to the schemes of closure that philosophy has sought to impose upon the former for the sake of its own salvation. [165]

So it was for the Pythagoreans, those metaphysical architects of Number, in whose time the diagonal of the square represented a limitation: a limitation correlative to an *expectation* ordained by the position of the integer within the operational unlimitedness [*l'illimité*] of a Principle. It was this unlimitedness of principle whose extra-mathematical – i.e. ideological – significance was evinced by the warping of the irrational, which determined the difference of another stratum.

An occasion to insist that there are no, and that there cannot be, crises *in* science, since science is the pure affirmation of difference.

But that a crisis in the (ideological) *representation* of science can induce a (positive) reconfiguration of science itself should not surprise us, given that the material of science is, *in the last instance*, ideology, and that an 'a priori' science by definition deals only with those aspects of ideology which represent it in the latter: a science continually breaking with its own designation in representational space.

It remains for us to address the 'hiatus' that allegedly separates Formalism from Intuition, the former failing to derive all the truth of the latter.

Let us begin by noting that the sense of 'intuitive' at issue here should be defined as the historical state of a science, both in the received, familiar intricacy of its density, and the binding, lawful circulation of its inscriptions.

Thus the problem relates an entirely coded scriptural artifice to the immanence of a historico-institutional discourse living off the abbreviations, equivocations, and univocal smoothing of an inoffensive mass of 'normal' signifiers legitimated by custom and practice.

Let us also note that the paradox of which Gödel makes implicit use occurs in ordinary language in the ancient lesson of the Liar: a statement that exhausts itself in stating its own falsity.

Thus, 'limitation' here comes down to the possibility of constructing a *predicate of non-derivability* (call it ~D) in a formal language and applying this predicate to a *representative* of the statement formed by this very application. Call this ~D(*n*), where *n* 'represents' ~D(*n*) in a sense that demonstrates the proof.[28]

Gödel's theorem does not then express a hiatus so much as a *reprise*, within the system's architectonic transparency, of certain ambiguities produced in language by the (ideological) concept of Truth. If one tries to make the Derivable subsume the True, then like the latter the former operates as a snare at the ungraspable juncture between science and its outside.

Gödel's theorem is thus one of formalism's *fidelity* to the stratifications and connectivities at work in the history of the sciences, insofar as they *expel* from the latter every employment of the True as (unlimited) principle.
[LINE BREAK]
But we have to engage with it if we are to understand it. To this end, we are going to give a largely intuitive but nevertheless complete and rigorous demonstration of the essential core of a limitation theorem.

This demonstration is taken from Raymond M. Smullyan's *Theory of Formal Systems* (Princeton, 1961).

Our exposition is governed by pedagogical considerations: in principle, this proof requires *no* particular mathematical knowledge – but this does not mean that it can be read inattentively.

The exposition proper will be accompanied by parenthetical commentaries [in italics], which reduplicate its meaning, in a way that is often dangerously ideological. Their function is didactic. [166]

The handful of further remarks prefaced by a !! are not necessary for an understanding of the deduction, but serve to suture it to the discourse of those readers who, knowing a minimum of mathematics, might be justifiably tempted to move faster than I will here.

The structure of the demonstration is as follows:

I) *Description of the System*

1. mechanism-2;
2. numbering of the inscriptions produced by M_2 (*g* function);
3. function of representation (φ function);
4. mechanism-3;
5. consistency.

[28] TN: 'Soit ~ D(*n*) où *n*, en un sens qui fait la preuve, "représente" ~ D(*n*).'

II) *Diagonalization Lemma*

1. diagonalization and W* classes;
2. Gödel statements;
3. representation of a class of numbers by a predicate;
4. diagonalization lemma.

III) *Condition for the Existence of an Undecidable Statement.*

(In what follows, results will be annotated according to this table: if, for example, the condition of consistency is evoked, it will be noted: (I,5)).

I Description of the System

1) ***Mechanism-2***

Let E designate the production of a mechanism-2 (a syntax), i.e. the set of well-formed expressions of a logical system.

We will suppose that the following inscriptions [*écritures*], among others, figure in this production:

— Predicates *p*, the set of which we will call P.
— Closed statements, the set of which we will call S.

(*Let us note right away that these designations pertain to this demonstration's semantic legibility. But purely set-theoretical data would suffice here:* E, P $\subset$ E, S $\subset$ E.)

2) Numbering the expressions

We are now going to give ourselves the set of integers, N, in its 'intuitive' sense, i.e. one that is recognizable to anyone familiar with the arithmetical tradition: 1, 2, 3, and so on.

And we are going to suppose that we have numbered all of the expressions in E. In other words, to every inscription $e \in$ E there corresponds an integer, noted $g(e)$; in addition, we will suppose that, reciprocally, every integer corresponds to one, and only one, expression in E.

!! We therefore posit the existence of a bi-univocal mapping *g* of E onto N.

(*This step is essential; it inscribes the inscriptions of* M_2 *as a* denumerable *infinity. If, moreover, our system 'formalizes' arithmetic, then it will be able to 'talk' about its own inscriptions, by 'talking' about the numbers which correspond to those inscriptions through the numbering function.*) [167]

3) Function of Representation

(*We now want to make sense of the idea that our system is a strong one, i.e. that it operates on the inscriptions of arithmetic.*

Intuitively – and vaguely – this could be taken to mean that the inscription formed by an expression and a number *is a new expression, internal to the system. Or, if one prefers: that in applying or 'mapping' an expression onto a number, one obtains an inscription within the system, which thereby 'talks' about numbers.*)

We are going to assume that there exists a function φ, which we will call 'the representation function', which associates the pair formed by an expression and an integer with another expression. Thus:

$$\varphi(e, n) = e'$$

(with $e \in$ E, $n \in$ N, $e' \in$ E).

!! φ is therefore a mapping of E X N into E. We have:

$$(\mathrm{E} \times \mathrm{N}) \underset{\varphi}{\rightarrow} \mathrm{E} \underset{g}{\rightarrow} \mathrm{N}$$

(*The most interesting case is one where the expression e is a predicate: to put it intuitively, φ(p,n) can 'express' the mapping from the 'property' p to the number n. And we should then be able to ask, without ambiguity, whether or not the expression φ(p,n) is true; whether or not the number n has that property. It should therefore be possible to consider the expression φ(p,n) as* complete*, i.e. as producing a univocally justifiable meaning for an evaluation.*)

We will posit that every expression φ(*p,n*) is a *closed statement* (belongs to S; see I.1): φ(*p,n*) ∈ S.

4) Mechanism-3

That a mechanism-3 (of derivation or demonstration) operates upon the closed statements means:

- That there exists in S a set of expressions which we will call *provable*. Let D be this set (D ⊂ S).
  (S – D) therefore represents the set of unprovable statements.
- That there also exists in S a set R of *refutable* expressions. (R is thus the set of the expressions whose negation is provable.) (R ⊂ S)

(*We have revealed the* two *conditions that characterize a mechanism-3: dichotomy* (*derivable and non-derivable,* D *and* S – D)*, and correspondence via negation, which gathers together the expressions whose negations can be derived* (R).

Gödel's problem, accordingly, consists in knowing whether every unprovable *(closed) statement is* refutable*. Is it always the case that* (S – D) = R *?*

Our task is to establish the structural conditions that render this equation impossible.) [168]

5) Consistency

We want there to be no encroachment of the provable upon the refutable, which would amount to a contradiction. No expression should, therefore, simultaneously belong to both D and R: the system will be said to be *consistent* if the intersection of these two sets is void, i.e. if D ∩ S = Ø.

!! At the level of generality at which we are operating, it is clearly possible to abstain from all interpretation, and say that we assume the following:

– E, P ⊂ E, S ⊂ E, R ⊂ S, D ⊂ S.
– R ∩ D = Ø.
– N, $\mathrm{E} \underset{g}{\rightarrow} \mathrm{N}$ (where *g* is bi-univocal onto).
– (E X N) → E, with (∀*p*)(∀*n*)[(*p* ∈ P, *n* ∈ N) → φ(*p,n*) ∈ S].

φ

II Diagonalization Lemma

(1) Diagonalization and W* *Classes*

Among expressions of the form $\varphi(e,n)$, there are some which are particularly interesting: those in which the number n is precisely the number $g(e)$ that 'numbers' [*numérote*] (see I.2) the expression e.

The expression $\varphi(e, g(e))$ is called the *diagonalization of e*.

(*This is the core of the proof: we 'map' the expression to the number that 'represents' it*.)

Let us now consider a set of expressions of E, any set whatsoever: call it W (so that we simply have $W \subset E$).

We will suppose that there are within W diagonal expressions of the form $\varphi(e, g(e))$. We are going to associate the class of *numbers* W* to the set *of expressions* W; this class W* will encompass all the numbers that number expressions whose diagonalization is in W.

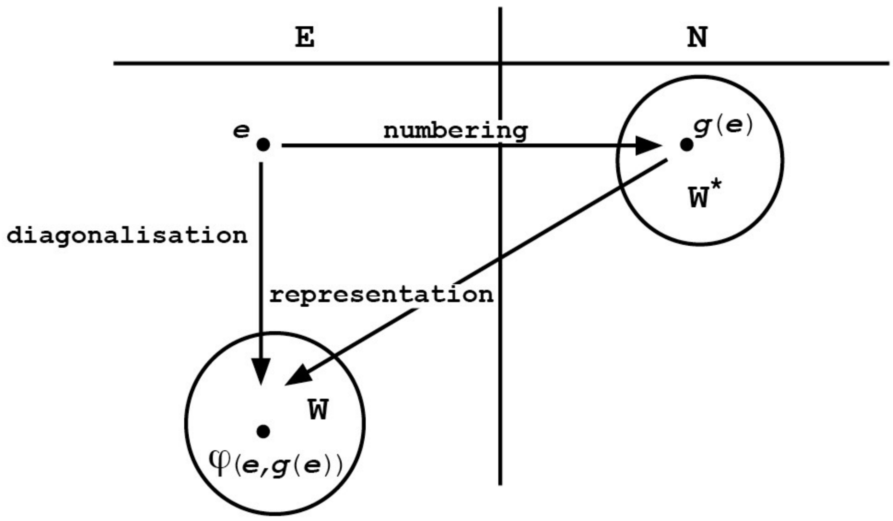
For any number n, to belong to W* means that there exists an expression e, such that:

(a) $g(e) = n$ (n 'represents' e);
(b) $\varphi(e, g(e)) \in W$ (the diagonalization of e is in W).

Or again, using the classic symbol $\leftrightarrow$ for equivalence:

$$n \in W^* \leftrightarrow [n = g(e)] \text{ and } [\varphi(e, g(e)) \in W]$$

Naturally, if W contains no diagonal expressions, then W* is the empty class.
[169] We therefore have the following situation:



!! Let φ_d be the diagonal-function over E defined by: $\varphi_d(e) = \varphi(e, g(e))$. We then have: $W^* = g \circ \varphi_d^{-1}(W)$.

2) Gödel Statement for a Set of Expressions

The guiding idea is now to associate a set W of expressions with a (closed) *statement* which is such that its 'truth' depends on its position with respect to W – a statement, in other words, that is provable *if and only if it belongs to* W.

Such a statement (belonging to S, see I.1), call it G, therefore satisfies (recall that D is the set of provable statements) the statement:

$$G \in D \leftrightarrow G \in W$$

It is called a *Gödel statement for* W.

(*A Gödel statement for the set of expressions* W*, if it exists, is therefore a statement whose* demonstrability *is 'expressible' in terms of belonging to* W*. We have here a rough equivalent to what Gödel demonstrates – laboriously – in his system: the possibility of constructing in the latter the predicate 'provable in the system'.*)

(3) Representation of a Class of Numbers in the System

We will say that a *predicate* $p \in P$ (see I.1) represents a class of integers $A \subset N$, if we have:

$$\varphi(p, n) \in D \leftrightarrow n \in A$$

(*The 'mapping'* [application] *of p onto the number n yields a provable statement if and only if this number belongs to the class* A*. This is a very formal elaboration of the following intuitive idea: the property p belongs only to numbers of the class* A*. Or: the class* A *is the extension of the concept p.*) [170]

(4) Diagonalization Lemma

We are going to demonstrate the following proposition: *If a class of integers* W* *is representable in the system by a predicate, then there exists a Gödel statement for the set of expressions* W.

Let *p* be the predicate which represents W*. By definition (from the preceding paragraph):

$$\varphi(p, n) \in D \leftrightarrow n \in W^*.$$

In particular, for $n = g(p)$

(i) $$\varphi(p, g(p)) \in D \leftrightarrow g(p) \in W^*.$$

(*The* diagonalization *of the predicate p is* provable *if and only if the* numbering *of p belongs to* W*.)

But (see II.1), the very definition of the class W* is:

(ii) $$g(p) \in W^* \leftrightarrow \varphi(p, g(p)) \in W.$$

By *juxtaposing the equivalences* (i) *and* (ii), we obtain (by substitution of a term equivalent to the one on the right in (i), or, if one prefers, by applying the transitivity of equivalence):

$$\varphi(p, g(p)) \in D \leftrightarrow \varphi(p, g(p)) \in W.$$

Here we recognize (see II.2) the definition of a Gödel statement for W: $\varphi(p, g(p))$ is this *statement* (and it is indeed a closed statement since – (see I.3) – we have postulated that when p is a *predicate,* the expression $\varphi(p, n)$ always belongs to S).

(*What have we demonstrated? That if a class* W* *(of numbers) is represented by a predicate in the system, then the* diagonalization of that predicate *is a Gödel statement for the set of expressions* W.

Let us take one more step in the (ideological) description *of this result.*

Let W *be any set of expressions whatsoever. Suppose that* W *contains diagonal expressions (expressions 'mapped' onto the number that represents them in the numbering of expressions). Consider then the set of numbers that number these diagonal expressions. This set is* W* *(see the schema in* II.1*).*

To say that W* *is represented in the system is to say that there exists a predicate whose 'meaning' is: 'to be a number that represents a diagonal expression contained in* W*'.*

We now diagonalize this predicate ('mapping' it to its own numerical representation). We then obtain a statement, *the meaning of which would be something like: 'The number that represents the predicate, "to-be-a-number-that-represents-a-diagonal-expression-contained-in-*W*", is itself a number that represents a diagonal expression contained in* W.'

It is this statement that is not provable unless it belongs to W*; it is therefore a Gödel statement for* W.

Here one will recognize the underlying structure of the diagonal processes that, ever since Cantor, have provided 'foundational' mathematics with its principal instrument: the construction of a statement that affirms its own belonging to a group of expressions which this statement also represents or designates.) [171]

III Condition for the Existence of an Undecidable Statement

The guiding idea for the completion of the argument is very simple: we are going to apply the diagonalization lemma to the class R of *refutable* statements. And we will thereby quite easily obtain *Gödel's Theorem*: *if* R* *is representable (in the sense of* II.3*), then there exists a statement that is neither provable nor refutable (one which belongs to neither* D *nor* R*).*

If R* is representable, then there exists a Gödel statement for R (in keeping with the diagonalization lemma). Let G be this statement. By definition (II.2):

$$G \in D \leftrightarrow G \in R$$

(i.e. G is provable if and only if it is refutable...)

But $D \cap R = \emptyset$ (i.e. D and R have no elements in common: hypothesis of consistency, I.5).

Consequently G belongs to *neither* D *nor* R: it is an *undecidable statement.*

(*What does the initial hypothesis signify: '*R* *is representable'? It signifies that there exists in the system a predicate whose 'meaning' would be: 'to be a number that represents a refutable diagonal expression'.*

As for the Gödel statement for R *– the undecidable statement – we know, by the demonstration of the lemma, that it is none other than the diagonalization of the* *predicate that represents* R*. *It is therefore a statement whose 'meaning' is something like:*

'The number that represents the predicate, "to-be-a-refutable-diagonal-expression," itself represents a refutable diagonal expression.'

Here one will recognize a kinship with the 'intuitive' argument of the Liar.)

This demonstration foregrounds the kernel of Gödel's discovery: if one can construct a predicate of refutability in a system, its 'mapping' to diagonal expressions results in the undecidability of a certain class of statements.

This demonstration also sheds light on the zigzagging movement that the proof co-ordinates 'between' the formal system and the 'intuitive' theory of integers: it is the representation (the numbering) of expressions that makes the diagonalization possible. Inversely, it is the construction of the W* classes in N that, predicatively 'reprised' in the system, renders possible the crucial demonstration of the lemma. In our vocabulary, we will say that this proof operates upon the connectivities between strata, which authorize trajectories and correspondences.

The scrupulous complexity of Gödel's demonstration, as compared to Smullyan's, has to do with the fact that the former has to establish the representability of R* for a *determinate* system (basically that of Russell and Whitehead's *Principia Mathematica*).

But the very general point of view adopted by Smullyan clearly draws out the structural and positive character of Gödel's discovery. As so often happens in mathematics, the latter deploys a network of conditional *constraints*: by prescribing for our system both consistency ($D \cap R = Ø$) *and* a 'strong' representative capacity [172] (the class of numbers R* is 'designated' by an expression in E), we can construct a 'remainder' in the set of statements, thereby showing that the disconnected sets D and R do not *overlap* with S.

We should bear in mind that the concepts of representability, consistency, disconnectedness, numbering, etc., have been given a mathematical assignation here, and retain none of their empirical or philosophical connotations. The concept of 'representative', in particular, is one that we have used in a figurative sense only, in stead and in place of that which it covers: *functions* (g and φ), defined in a perfectly classical way.

Thus, Gödel's result is peculiar and dramatic only with respect to a semantic saturation which *imposes upon* the discourse of science an *ideological expectation*.

Whoever poses to logic questions that are not problems runs the risk of registering as *resistance* what is in fact simply the deployment of those regional constraints through which this science's artificial object *occurs*.

In this way we re-encounter the articulated dialectic of science and ideology. In terms of the problem that concerns us here, its stages are the following:

I) The existence of a historical mathematics (namely 'intuitive' arithmetic), one that is open in principle (indefinitely stratified signifier).

II *a*) The ideological re-presentation of this existence as the trans-mathematical norm of thoroughly controllable rationality (ideological destratification of the mathematical signifier).

II *b*) The posing of a question to mathematics about their conformity to this ideological norm, in the form of the axiomatic and formalist *intention*, whose goal is to display a *well-founded transparency* (ideological motivations of Frege and Russell).

III) Break: the mathematical treatment of this ideological re-presentation of mathematics via the actual *construction* of formal systems that 'represent' historical arithmetic (*Principia Mathematica*).

IV *a*) The ideological re-presentation of this break: formal systems conceived as trans-mathematical norms of rational closure. The *idea* of a nomological system (Husserl).

IV *b*) The posing of a question to mathematics about their absolute conformity to the ideological norm of closure: this is the meta-mathematical *intention*, relative to the *internal* demonstration of a system's consistency (Hilbert).

V) Break: the mathematical treatment of ideological re-presentation via the actual *construction* of a mathematical metamathematics (arithmetization of syntax).

Gödel's Theorem: the structural stratification of the mathematical signifier does not answer the 'question' of closure.

VI) Ideological re-presentation of this break: Gödel's Theorem is experienced [*vécu*] as a *limitation* relative to the normative expectation.

Ideological exegesis of this 'limitation' as:

– openness of speech and concealment of being (Ladrière);
– finitude;
– splitting, suture;
– ...

VII) Break: the general theory of the limitation-effect, positively conceived as a structural dimension [*instance*] of certain mathematical objects (Smullyan's epistemological truth). [173]

The epistemological upshot of this convoluted adventure reminds us that mathematics *operates upon its own existence as it is designated in ideology*, but that this operation, conforming to the specific constraints of a science, takes the form of a *break*, such that the (ideological) questions which make up the *material* upon which mathematics carries out its working reprise [*reprise oeuvrante*] find *no answer* in the latter.

By coming to figure in the space of mathematics' problematic, the ideological image of this science can henceforth only be *misrecognized* by whoever proffered it. In the shift from material to product, mathematics obeys rules of existence that *nothing* in the material could have indicated.

This makes it quite clear that science is science of ideology, and is even science of the ideology of the science of ideology, and so on, as far as you like. But ideology never finds itself in it.

Such is the law of the alternating chain in which what is known as 'the progress of science' consists: it is not because it is 'open' that science has cause to deploy itself (although openness governs the *possibility* of this deployment); it is because ideology is incapable of being satisfied with this openness. Forging the impracticable image of a closed discourse and enjoining science to submit to it, ideology sees its own order returned to it in the unrecognizable form of the new concept; the reconfiguration through which science, treating its ideological interpellation as material, ceaselessly displaces the breach that it opens in it.

Let us take stock here – this time in close proximity to Lacan – of the ridiculousness of the claim that progress is motivated by the 'intention' of discovery.

With regard to Gödel's theorem, and the limitative connotation that (in the wake of the irrational, the negative, and the imaginary numbers) heralded it, we should remember that science advances *precisely* through those who, putting to it the question of its obstruction [*arrêt*], are engaged in desperately ordering the place where it may be recognized that this question, however much it may be *reprised*, is not even understood.

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