Yves Duroux

Psychology and Logic[1]

*[**Editorial note: this manuscript file will be replaced by a definitive pdf in early 2013**]*

[31] My presentation is based on a reading of Frege's *Grundlagen der Arithmetik* (Breslau, 1884).[2]

The object of this investigation is what we might call the natural progression of whole numbers. We can study the properties or the nature of number. But the properties of number conceal its nature.

By a property of number I mean what mathematicians do in a domain delimited by Peano's axioms.[3] The properties of whole numbers are drawn from these axioms. But in order for these axioms to function and to produce these properties, it is necessary that a certain number of questions are excluded from the field, questions whose terms, given as self-evident, concern the nature of the number. There are three such questions:

1. What is a number? (Peano's axiom takes it for granted that one knows what a number is).
2. What is zero?
3. What is the successor?

It is on the basis of these three questions that responses regarding the nature of whole numbers can vary.

For my part, I will look at the way Frege, criticizing a tradition, articulates his response. The whole of this critique [32] and of this response, as I will present them here, will constitute the foundation [*butée*] on the basis of which Jacques-Alain Miller will develop his presentation.[4]

If zero is not reflected in a function that is different from the other numbers (if it is not taken as a point from which a succession is possible), if zero is not given a prevailing function – then the two other questions may be stated as follows:

1. How do we pass from a collection of things to a number that is the number of these things?
2. How do we pass from one number to another?

---

[1] This is the account of a presentation given on 27 January 1965 at the seminar of Dr. Jacques Lacan [at the Ecole Normale Supérieure]; it has not been subsequently revised by the author. TN: First published as Yves Duroux, 'Psychologie et logique', CpA 1.2 (January 1966): 31-36. Translated by Cécile Malaspina, revised by Peter Hallward.

[2] TN: Gottlob Frege, *The Foundations of Arithmetic* [1884], trans. J.L. Austin (NY: Harper, 1960). In the fourth chapter of this book (sections 72-80), Frege presents a logical construction of the series of whole or 'counting' numbers, derived from the definition of zero, the number one, and the ‘successor function’ (denoted by the + sign).

[3] TN: In his *Principles of Arithmetic Presented by a New method* (1889) Giuseppe Peano lists the axioms that determine the sequence of whole numbers. Among other things, these axioms characterise the relation of equality (as reflexive, symmetric and transitive), and they assert that every number has just one successor, that no two numbers share the same successor, and that 0 is a number that is not the successor of any other number.

[4] TN: This presentation was subsequently published as 'Suture' (CpA 1.3).

These two operations, one of collection [*rassemblement*], the other of addition, are treated by a long empiricist tradition as operations that can be referred back to the activity of a psychological subject. This whole translation plays on the word *Einheit,* which in German means 'unit [*unité*]', such that it is from a *play on words* on this word that a series of ambiguities regarding the functions of successors and of number becomes possible.

An *Einheit* is first of all an undifferentiated and undetermined element of any given set [*ensemble*]. But an *Einheit* can also be the name *One* [*Un*]*,* name of the number 1.[5]

When we say one/a [*un*] horse and one horse and one horse, the *one* can indicate a unit, that is to say one element in a set where '3' horses are posited, one beside the other. But as long as these units are taken as elements and gathered together in the collection, we absolutely cannot infer that there is thus a result to which the number 3 can be attributed – unless we impose by force this designation on the collection.

In order to be able to say one horse and one horse and one horse = three horses, we must proceed on the basis of two modifications. It must be the case:

1. that the *one* [*un*] is conceived as a number
2. that the *and* is transformed into the sign +.

But of course, once we have carried out this second operation, we still won't have explained anything: we will [only] have posed the real problem, which is to know how 1 plus 1 plus 1 make 3, once we no longer confuse the number 3 with the collecting of three units.

The source of the problem is that the recurrence [*retour*] of number brings with it a radically new signification, which is not the simple repetition of a unit. How can this return of the number understood as the sudden emergence of a new signification be thought, so long as we haven't resolved the problem of the difference between equal elements [33], posited one beside the other, and their number?

A whole empiricist tradition contents itself by relating the emergence of a new signification to a specific activity (a function of inertia) of the psychological subject, which would consist in adding (along a temporal line of succession) and naming.

Frege cites a number of important texts that all come down to promoting imaginary operations: collecting, adding, naming. To support these functions, which mask the real problem, one must *suppose* a psychological subject that operates and states them. If the real problem is to discover what is specific to the sign + and to the *successor* operation, we must tear the concept of number away from psychological determination.

Here is where Frege's original enterprise begins in its own right. This reduction of the psychological proceeds in two stages:

1. Frege enacts a separation in the domain of what he calls the domain of *Vorstellungen* [representations]: on one side he puts what he calls psychological, subjective *Vorstellungen,* and on the other side what he calls objective *Vorstellungen.* This separation aims to *efface* all reference to a subject and to treat these objective representations according to laws that deserve to be called logical.

In these objective representations the *concept* and the *object* must be distinguished. Close attention must be paid to the fact that concept and object cannot be separated; the function that Frege assigns them is no different from the function of the predicate with respect to a subject, it is nothing other than a monadic relation (Russell), or a relation of function to argument.

[5] TN: The French indefinite article 'un/une' can be translated either as 'a' or as 'one'.

2. Building on this first distinction, Frege effects a second one that allows him to relate a number, no longer to a subjective representation as the empiricist tradition tried to do, but to an objective representation, which is the concept. The diversity of possible numerations cannot be founded on a diversity of objects. It is simply the indication of a substitution of the concepts to which number applies.

Frege gives a rather paradoxical example. He takes the sentence: 'Venus has no moon [*aucune lune*].' To what should we attribute the determination 'no'? Frege says that we do not attribute 'no' to the object 'moon' – and with good reason, since there is none. Nevertheless *zero* is a numeration; we thus attribute it to the *concept* 'moon of Venus'. The concept 'moon of Venus' is related to an object which is the object 'moon', and this relation is such that there is no moon. [34]

It is on the basis of this double reduction that Frege obtains his first definition of number (the various different definitions of number only serve to ground the successor operation). First definition of number [*le nombre*]: number belongs to a concept.

But this definition is still incapable of giving us what Frege calls an individual number, that is to say a number that possesses a definite article: *the one, the two, the three,* which are unique as individual numbers (there are not several *ones,* there is only *one one, one two*).

We still have nothing that would allow us to determine whether that which is attributed to a concept is this number which is the unique number preceded by the definite article.

To help us understand the need for another way of reaching this individual number, Frege takes another example of planets and their moons, and this time it is: 'Jupiter has four moons.'

'Jupiter has four moons' can be converted into this other sentence: 'the number of Jupiter's moons is four.' The *is* that links the number of Jupiter's moons to four is absolutely not analogous to the *is* of the sentence 'the sky is blue': it is not a copula, it is a function of equality. The number *four* is the number one must posit as equal (identical) to the number of Jupiter's moons; to the concept 'Jupiter's moons' is attributed the number *four.*

This detour obliges Frege to posit a primordial operation that allows him to link numbers to a purely logical relation. This operation – I won't describe it in detail here – is an operation of 'equivalence'[6], a logical relation that enables one to order objects or concepts in a one-to-one correspondence (this 'or concepts' need not worry us in so far as, for Frege – at least at this stage of his thought – each relation of equality between concepts equally orders the objects falling under these concepts according to the same relation of equality).

Once this relation of 'equivalence' has been posited one can arrive at a second, the genuine definition of the number: 'the number that belongs to the concept f is the extension of the concept equivalent to the concept F.' [35]

This means: we have posited a determined concept F; we have determined, through the relation of equivalence, all the equivalences of this concept F; we then define the number as the extension of this concept equivalent to the concept F (all the equivalences of the concept F).

Frege's thinking will then proceed on the basis of a machine that we might arrange along two axes: a horizontal axis upon which the relation of equivalence comes into play, and a vertical axis which is the specific axis of the relation between concept and object (we can always, from the moment we have a concept, transform it into the object of a new concept, since the rapport between the concept and the object is a purely logical rapport of relation). It is on the basis of his relational machine that Frege now

[6] Or again of 'identity'.

claims to demarcate [*cerner*] the various numbers, the individual numbers, which in a certain way he places at the end of his investigation, as the crowning of his system of equivalence. To demarcate the various numbers comes down to defining zero and the successor.

In order to obtain the number zero, Frege forges the concept of 'not-identical to itself', which is defined by him as a contradictory concept, and he declares that, to whichever contradictory concept (and he evokes the commonplace contradictory concepts of traditional logic, the square circle or the wooden metal), to whichever concept under which no object falls, is attributed the name: 'zero'.[7] Zero is defined by logical contradiction, which guarantees non-existence of the object. There is a referral from the non-existence of the object, which is certified, decreed (since one says that there is no centaur or unicorn), to the logical contradiction of centaur or unicorn.

The second operation, which allows us to generate the entire series of numbers, is the successor operation. Frege defines at one and the same time the *one* and the successor operation.

For the successor operation I will give only Frege's definition, which he posits before the one; then I will show how he can only avail himself of this successor operation because he provides himself with this relation of one to zero.

The successor operation is defined simply as follows:

We say that a number immediately follows another number in a series if this number is attributed to a concept under which falls an object (x), and that there is another number (this is the number which the first number follows such that it is attributed to the concept 'falling under the preceding concept, but not identical to (x)').

This definition is purely formal. Frege grounds it, immediately afterwards, by defining the one. The one is defined on the basis [36] of the concept 'equal to zero'. What object falls under this concept? The object zero. Frege then says: '1 follows 0 in so far as 1 is attributed to the concept "equal to 0".'[8]

So: the successor operation is generated by a double play of contradiction in the passage from zero to one. We might say, without going far beyond Frege's own field, that the reduction of the successor operation is accomplished by the operation of a double contradiction. Zero is given as contradictory; the passage from zero to one is given as a contradictory contradiction. The motor that animates succession in Frege is purely a negation of the negation. The apparatus that enables the definition of number functions very well. But is it capable of answering the following question: 'how, after 0, is there 1'? I will not question the legitimacy of the operation here. I will leave Jacques-Alain Miller to attend to this.

I would just like to make two remarks:

1. For the empiricists, as for Frege, the name of [a] number (which Frege calls an individual name) is only obtained, in the end, by *a coup de force*, like a seal that the sealed might apply to itself.

2. For Frege, as for the empiricists, number is always captured by an operation whose *function is to fill up* [*de faire le plein*][9]*,* through a collecting, or by this operation that Frege calls one-to-one correspondence and which has precisely the function of exhaustively collecting a whole field of objects. The activity of a subject on the one hand, and the logical operation of equivalence on the other, have the same function. We will have to draw the consequences of this.

---

[7] TN: '0 is the number that belongs to the concept "not identical with itself"' (Frege, *The Foundations of Arithmetic* §74).

[8] TN: '1 is the number which belongs to the concept "identical with 0"' (Frege, *The Foundations of Arithmetic* §77).

[9] TN: The phrase '*faire le plein*' means to fill up a car at a petrol station.